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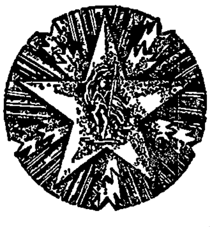

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Za izdavača: Dr **Petar Radičević,** dipl. inž., sekretar Saveza inženjera i tehničara Jugoslavije

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Pregled stanja u analizi elastičnog ponašanja ravnih grednih sistema

Dr Ing. *RICHARD BAREŠ*, Institut of Theor. and Appl. Mech., Praha, Čehoslovačka

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Zahvaljujući pogodnim statističkim efektima koje poseduju, sistemi ravnih greda su našli široku primenu u praksi, mada iznalaženje stvarne raspodele naprezanja u njima, kada su opterećeni spoljnim opterećenjem, zadaje velike teškoće zbog njihove velike statičke neodređenosti. U istoriji mehanike je malo konstrukcija kojima je posvećena tolika pažnja kao ovim sistemima. Bibliografija koja se odnosi na njih je ekstremno bogata: obrađeno je na desetine hipoteza, postupaka i metoda sa namerom da se objasni rad pojedinih članova sistema i smisli pogodan metod za analizu. Veliki broj disertacija, doktorskih teza i istraživačkih radova objavljenih na fakultetima i istraživačkim institutima po celom svetu bavio se analizom ovih sistema. Međutim, samo neki od tih radova su postigli svoj cilj. Kod većine metoda proračunski deo je toliko veliki da postupak nije mogao naći veću primenu, bilo zbog nedostatka vremena bilo zbog nemogućnosti da se matematički aparat dovede do stvarne primenljivosti. I oni koji su više ušli u problem ponekad su se zadovoljavali manje ili više aproksimativnim proračunima. Takav je bio kraj jednog niza ambicioznih metoda.

Teorije koje pokrivaju elastično ponašanje ravnih sistema greda mogu se uopšteno podeliti u tri grupe:

a) Teorije koje opisuju konstrukciju u njenom realnom obliku pokrivajući ponašanje svakog pojedinog elementa konstrukcije pomoću sistema preopširnih linearnih, simultanih jednačina. Broj ovih teorija je vrlo visok i one su primenljive kako u direktnoj tako i relaksacionoj analizi uz pomoć računara.

b) Teorije koje zamenjuju realan sistem greda prostijim sistemom čija je krutost ekvivalentna sjedinjenoj poprečnoj krutosti glavnih greda. Ponašanje konstrukcije je pokriveno skupom simultanih diferencijalnih jednačina. Samo u najprostijim slučajevima računar može biti izostavljen.

c) Teorije koje zamenjuju realan sistem pojedinačnih elemenata elastično ekvivalentnim sistemom jednako raspodeljenim u oba pravca. Analiza može koristiti širok matematički repertoar upotrebljavan u analizi kontinuuma, sa ili bez pomoći računara.

Teorije pod a) i b), koje se mogu okarakterisati kao statika štapova, bile su obrađivane počev od 1889. god. kada je Engesser publikovao svoj rad, pa do otprilike 1945. god. Renesansa njihovog razvoja pada posle 1960, kada počinje da se uvodi nova tehnologija računanja. Različiti numerički postupci, upotrebljavani do 1947, temeljno su pomenuti u obimnom Jansenniusovom radu [1], dalji doprinosi se mogu naći u radovima Beera [2] i Jägera [3]. Oni svedoče o ponovljenim pokušajima da se aproksimativnim metodama nađe najpreciznije rešenje problema.

U analizi metodom deformacione energije nepoznate su sile koje dejstvuju između glavnih i poprečnih greda [4], u slučaju oslonaca sa trnom ili kod krutih spojeva to su vezne sile i momenti u dva pravca [3], [5]. Broj nepoznatih je n ili 3n, ako je n broj spojeva. Hilal [18] je pokazao da čak i u slučaju krutih spojeva vezne sile mogu biti upotrebljene kao nepoznate veličine, koje dejstvuju na podužne i poprečne grede kao dva nezavisna sistema kontinualno, elastično vezanih greda. Ali rešenje zahteva prethodno određivanje stalnih tačaka oba sistema greda.

Rešavanje 3n jednačina se može izbeći ako se može naći sistem spoljašnjeg, takozvanog sopstvenog opterećenja pod kojim se rešetka vertikalno savija u jednostavnu formu (linije savijanja svih podužih i/ili poprečnih greda su slične). Ovi sistemi opterećenja se moraju kombinovati tako da daju stvarno stanje opterećenja. Metoda sopstvenih opterećenja je generalizacija metode Fourierovih redova i u saglasnosti je sa uopštenom metodom rešavanja zasnovanom na primeni ortogonalnih funkcija, često upotrebljavanoj u matematičkoj mehanici. Tu metodu su prvi objavili Bleich i Melan [6], koji su primenili diferencijalnu analizu; kasnije su je Koch [7] i Thoms [8] razvili na drugi način; precizno rešenje su takođe dali Melan i Schindler [9], Homberg [10], Trost [53] i drugi. Očigledni nedostatak metode sastoji se u činjenici da se, kao prethodno, mora odrediti n sistema sopstvenih opterećenja. Određeno pojednostavljenje je moguće ako je održana proporcionalnost između pomeranja čvorova i spoljašnjih

Adresa autora: dr Richard Bareš, Institut of Theor. and Appl. Mech., 12849 Praha 2, Vyšehradska 49, ČSSR

sila koje dejstvuju samo po podužnim ili poprečnim gredama.

Ako je broj poprečnih greda beskonačan, sopstvena opterećenja su sinusoidna, što omogućava prilično dobar analitički postupak, kao što je pokazao Timoshenko [14] za slobodno oslonjene poprečne grede a Hetenyi [15] i Weber [16] za poprečne grede sa slobodnim krajevima. Hondermarcq [17] je nastavio sličnim pravcem zamenjujući beskonačan broj poprečnih greda kontinualnom pločom oslonjenom na paralelne grede. Metoda Pipparda i De Waella [33] koristi uniformnu podelu poprečne krutosti; autori su dobili sistem diferencijalnih jednačina čije je rešenje suviše kompleksno da bi bilo praktično primenljivo. Homberg [11], Clarkson [12], Biermann [13] i drugi objavili su tabele za neke tipove roštilja zasnovane na primeni metode deformacione energije.

Kod »ugib — nagib« (»slope — deflection«) metode nepoznate su vertikalno pomeranje i uglovne deformacije čvorova. Ovu metodu je prvi primenio Ostenfeld [18] za slobodne oslonce sa trnom. Genntner je rešavao problem sa krutim spojevima a zadovoljavajuće rešenje je takođe postavio Palotás [20].

Razrađen je takođe, uz pomoć metode deformacione energije i »ugib — nagib« metode zajedno, jedan pogodan kompjuterski postupak, naročito ako je analiza aranžirana u matričnom obliku U tom smislu mogli bi se koristiti i radovi koji rešetkastim sistemom štapova modeliraju elastični kontinuum. Prvu monografiju ove vrste objavio je Hrenikoff [21] (kvadratni model roštilja sa konstantnim Piossonovim brojem 1/3). Za jedan opštiji model ravnih greda rešenja su dali Newmark [22], Ang i Newmark [23] i Yettram i Hussain [24]. Druge specijalne slučajeve su istraživali Christensen [25], Lightfoot [26], i Hudson i Matloch [27]. Willems i Lucas [28] i drugi [29], [30], [31] objavili su matrični oblik jednačina uglovnih deformacija, to isto su uradili Eisenmann i ostali [32] 1962. god. za veliki broj štapova. Na taj način organizacija računanja može lako biti aranžirana u sistem tako da se sa programiranjem može započeti odmah. Metoda opet pretpostavlja da je svaki štap izložen jediničnim ugaonim deformacijama u dva pravca i jediničnim pomeranjem u trećem. Sile izazvane jediničnim deformacijama su takozvani koeficijenti krutosti i obrazuju matricu krutosti svakog pojedinog člana rešetke. Rotirajući ove matrice u jednom jedinstvenom sistemu koordinata dobija se, iz uslova kompatibilnosti deformacija čvorova, matrica celog sistema štapova. Rotirane matrice su priređene unapred tako da za svaki pojedini elemenat treba samo zameniti brojne vrednosti krutosti, dimenzija i ugla rotacije matrice. Precizna numerička analiza roštilja metodom deformacione energije i aranžman koji zahtevaju elektronski računari takođe su obrađeni u jednom broju radova Ulickog, na primer u [54]. Uprkos činjenici da matrična organizacija računanja poseduje gore pomenuta preimućstva i činjenici da se brzi računari mogu lako dobiti, ove metode su podešene pre pri analizi nekih posebnih slučajeva nego upotrebi u tekućoj praksi.

U nameri da se izbegne sistem sa velikim brojem jednačina izmišljene su različite relaksacione metode, kod kojih se ranije jednačine, koje uopšte nisu ni napisane, rešavaju postepenom aproksimacijom. Početna tačka je rešetka; njeni se čvorovi postepeno oslobađaju (pomeranjem i rotacijom u oba pravca) na način koji je sugerirao Cross u analizi okvira. Za čvorove koji nisu kruti ovakvu metodu su razradili Southwell [34], Jansonius [1] i drugi, a za krute spojeve Van Russelt [36] i Jansonius [1]. Lazarides [7] je bio u stanju da primeni ovu metodu na istraživanje prethodno napregnutih roštilja sa različitim tipovima oslanjanja.

Sve pomenute metode moraju, zbog utroška vremena računanja, da sadrže numerički određene uticajne površine; pa će dobijanje vrednosti unutrašnjih sila biti prilično teško i netačno, što takođe u potpunosti degradira tvrđenje o visokoj tačnosti metode.

Pokušavano je da se problem reši izvesnim brojem približnih metoda koje analiziraju samo jedan deo roštilja, zamenjuju realne elemente idealizovanim i uvode uprošćenja a priori.

Bechyně [38] je, na primer, ispitivao roštilj upotrebljavajući jednačine pet momenata; u njegovom slučaju poprečne grede su bile elastične i oslonjene na elastične oslonce slične kontinualnim gredama.

Leonhardt [39] bira uprošćeno poprečno povezivanje takvo da su sve poprečne grede koncentrisane u jednu centralnu poprečnu gredu iste krutosti, transfer je bio zasnovan na eksperimentalnim opažanjima. Torzioni efekti nisu bili uključeni. Ovu ideju su takođe razvijali češki autori Pacholík [40], Brebera [41], Faltus [42] i Dašek [43]. Poslednja dvojica su rešila problem koristeći krutost poprečnih i podužnih greda u jednom utvrđenom odnosu, ili obrnuto, sa jednakim poprečnim presecima poprečnih i/ili podužnih greda za opterećenje koje je gradirano u skladu sa razvojem savijanja (tzv. idealna konfiguracija opterećenja ili harmonijska opterećenja); to im je omogućilo uvođenje jedne tzv. idealne poprečne grede. Međutim, rešenju nedostaje jasnoća i prilično ga je teško dobiti za višestruka stanja opterećenja i višestruke sisteme greda. Kollár [20] je usavršio metode koje koriste jednu jedinu elastičnu poprečnu gredu uvodeći torziju u razmatranje.

Dalje uprošćenje predstavlja zamenu elastičnih poprečnih greda beskonačno krutim poprečnim gredama, što je slučaj kod ranije pomenute Engesserove metode [45]. Tu metodu su kasnije preporučili Courbon [46], Mallet [47], Gaussens [53] a prema Gibschmanovom usmenom saopštenju slična aproksimativna rešenja (Proskurjakov od 1902 [49], Grigorjev [50] i Iljasevich) već su bila u upotrebi u sovjetskim projektantskim centrima.

Teorije pod c), u kojima je realan sistem individualnih elemenata reduciran na elastično ekvivalentan, kontinualno rasprostrt sistem, predstavljaju prelaz na mehaniku kontinuuma primenom diferencijalne jednačine ortogonalno anizotropnih ploča.

Fundamentalna diferencijalna jednačina savijanja tanke ploče bila je izvedena 1811 god od Lagrangea i Germain [55]. Poisson [72], u svom poglavlju o elastičnosti (1829), uvodi prvi put ispitivanje elastičnih ploča upotrebljavajući opšte jednačine elastičnosti; njegova formulacija nekih konturnih uslova je dozvoljavala rešavanje samo nekoliko prostih slučajeva. Godine 1850. Kirchoff [53], koristeći princip energije, izvodi jednačinu koja opisuje stanje i odgovarajuće konturne uslove reducirajući njihov broj na slobodnoj ivici. Prvu studiju greda koje leže na elastičnoj podlozi publikovano je godine 1867. Winkler [57]. Njegovu teoriju je proširio Zimmerman [58] 1888. god. Na polju ploča oslonjenih na elastičnu podlogu prvi doprinos dao je Hertz [59] godine 1881. koji je istraživao ploču pod koncentrisanim teretom. Hertz je, kao i Winkler pretpostavio da je reakcija podloge proporcionalna ugibu elastične ploče. Godine 1923. Westergaard [60] je proširio teoriju beskonačne, elastično poduprte ploče tako se se dobila jedna praktična metoda za analizu temeljnih ploča; ovaj metod je još u upotrebi. Timoshenko i Woinowski-Krieger [61] su objavili najskoriju diskusiju ovoga problema;

radilo se o centrično opterećenoj kružnoj ploči. Godine 1953. Livesley [62] je došao do formalnog rešenja problema polubeskonačne ploče i beskonačnog kvadranta. Tu sledimo dalje doprinose velikog broja autora, na primer Kerr [63], Gorbunov-Posadov [64], Klepnikov [65], Korenev i Czernigovska [66], Gorlov i Serebrjanyj [67], Zhemochkin i Siricyn [68] itd.

Fundamentalni radovi koji se tiču teorije anizotropnih ploča potiču od Hubera [72] godine 1921. Chwalla [44], Hearmon [69], Lekhnicky [70], Ambarcumyan [71] i drugi su dali pregled jednačina i različitih konstanti anizotropije.

Kako je precizna analiza napona i deformacija tankih ploča moguća samo u specijalnim slučajevima, upotrebljava se, kada se radi o opštem sistemu ploča, niz različitih aproksimativnih metoda rešavanja. Po potpunoj obradi klasičnih rešenja Lagrangeove jednačine (Navierovo, Levyevo, konačne razlike itd.) razvijena je, u kasnim pedesetim godinama, metoda konačnih elemenata koja obrađuje proizvoljne dvodimenzionalne probleme na primer od Argyrisa [73], Clougha [74], [75], Melosha [76] i Zienkiewicza [77]. Ova metoda deli kontinuum na geometrijski proste elemente napregnute unutrašnjim silama nepoznatih veličina, koje se određuju iz uslova geometrijske kompatibilnosti. Mahrain [78] je primenio ovu metodu na rebraste ploče. Proračun je relativno brz pod uslovom da je perfektno organizovan. Stepen tačnosti zavisi od oblika i veličine izabranog elementa i od stepena polinoma koji ga opisuje. Kada je program jednom ispitan onda proračun jednog stanja opterećenja sistema od, na primer, 1 000 jednačina uzima oko pola minuta. Umesto konačnih ravnih elemenata može se upotrebiti (konačan) štapni (linearni) elemenat, kao što su to pokazali, na primer, Eisenmann, Woo i Namyet [32]. Obe metode daju uporedive rezultate. Metoda ravnog elementa dopušta deformacije u spojevima zamenjenog sistema i matematičke relacije su najbolje definisane funkcijom deformacije. S obzirom da metoda linearnih elemenata koristi deformabilne elemente sa krutim spojevima to je bliža svakodnevno upotrebljavanoj proračunskoj tehnici (metodi deformacione energije). Prednost ove metode je da promena u ivičnim uslovima ne utiče na postupak ili na opisne jednačine, i mogu se lako uključiti razne spoljašnje ili unutrašnje nepravilnosti kao rupe, promene krutosti itd. Nedostatak ovih metoda, kao i svih metoda koje u širokoj skali koriste računar, je da je statičar prilično odvojen od realne konstrukcije, on ne može da kontroliše rezultat, izabere najekonomičniju alternativu itd. Otuda tako zvano »strogo« proračunavanje (uvođenjem više cifara u brojeve koji opisuju sistem) nije od koristi zahvaljujući činjenici da jedan broj idealizacija mora nužno ući u početne parametre. Situacija je ovde slična onoj u teorijama pod a) i b), gde su takođe, u rešavanju problema kontinuuma, našle širu primenu razne aproksimativne metode. Ovde su takođe, metode koje upotrebljavaju koeficijente raspodele (kao u metodama Loenhardta, Faltusa, Dašeka itd.) dokazale svoju prednost.

Guyon [79] je, pošavši od Masselina [80], izveo metodu deonih koeficijenata za konstrukcije sa nultom torzionom krutošću, a Massonnet [81] je, koristeći Hetenyiev pristup [15], proširio ovu metodu uvodeći i torzionu krutost konstrukcije. Osnovna pretpostavka obe ove metode je da je poprečna raspodela bilo koga opterećenja ista kao ona kod sinusnih opterećenja, što je Dašek [43] izveo za grede konstantnog poprečnog preseka. Detaljna eksperimentalna analiza je pokazala sasvim dobro slaganje sa stvarnošću, u prkos činjenici da u raspodeli unutrašnjih sila ima neslaganja, naročito u konstrukcijama sa pločom ili prednapregnutim konstrukcijama (npr. [82, 83, 84, 85, 86, 87]). Razlog ovome je da Guyonova pretpostavka o jednakoj raspodeli bilo koga opterećenja ima ograničenu važnost, ako je uticaj poprečne konstrukcije zanemaren.

Dalji razvoj pristupa koji koristi koeficijente raspodele stigao je do metode G-M-B (Guyon — Massonnet — Bareš) [89], koja omogućava laku analizu unutrašnjih sila pod proizvoljnim opterećenjem.

Godine 1971. objavili su Cusens i Pama [90] jednu modificiranu G-M-B teoriju, proračunavajući nove koeficijente za određivanje ugiba i podužnih momenata od jednog koncentrisanog tereta, kao zbir prvih devet članova reda kojim je opterećenje bilo predstavljeno. Na taj način oni su stvarno suzili doseg originalne metode. Oni su uveli uticaj Poissonovog koeficijenta samo u izraz za momente savijanja konstrukcije sa torzionom krutošću ploče (maksimalnom).

Na osnovu Pucher-ove metode singulariteta [92] Krug i Stein su analizirali uticajne površine za niz slučajeva krutosti (1961). Metoda omogućava proveru, projektovanje samo za konkretne slučajeve; priprema za računanje elektronskim računarom uzela je, prema Sattleru, nekoliko godina; jedna uticajna površina zahteva preko 5000 instrukcija. Na uticajnim površinama su takođe radili Bittner [93], Olsen i Reinitzhuber [94], Homber [10], Hoeland [95]. Odvojeno od toga egzistira niz rešenja Huberove jednačine [72] praćenih tabelama [96], [97], [98], [99] ili formulama [100], [101], [102].

Hossain (1975) [103] je predložio proširenje G-M-B metode na pločaste mostove sa srednjim osloncima, superpozicije efekata izolovanih reakcija, računatih iz uslova kompatibilnosti ugiba, sa efektima spoljašnjeg opterećenja. On je uveo numerički jednostavnije računanje uticajnih linija podešavanjem organizacije računanja.

Rowe [104] i Hossain [103] su ukazali na važnost uticaja Poissonovog koeficijenta, mada nisu uspeli da ga uvedu sa punom doslednošću.

Izvestan broj autora je dokazao da je nemoguće egzaktno rešenje konstruktivno anizotropnih sistema strogo zasnovano na Huberovoj jednačini, pošto rastojanje neutralne osovine menja svoj položaj zavisno od lokalnih krutosti. Efekat zida može značajno uticati na sva stanja napona gornjih vlakana konstrukcije. Neke metode uzimaju u obzir sve sile koje dejstvuju u ortotropnoj ploči; rešenje, međutim, vodi ka kapacijalnoj diferencijalnoj jednačini osmog reda, ili sistemu od dve parcijalne jednačine četvrtog reda. Zbog kompleksnosti graničnih uslova praktično je nemoguće dobiti rešenje [105 — 115] a ni primena računara ovde ne može ništa da izmeni. Primena ovih postupaka zahteva različita uprošćenja pretpostavki, koja više nego potiru svaki doprinos višem stepenu tačnosti originalnog postupka.

Odvojeno od traženja pogodnog načina rešavanja Huberove jednačine, niz autora je učinio veliki napor da odredi što je moguće bliže ulazne veličine, naročito krutosti na savijanje i torziju. Naročito se u razvoju metoda zasnovanih na primeni računara oseća velika disproporcija između tačnosti računanja i nezadovoljavajuće definisanih ulaznih parametara.

Određivanje torzione krutosti ploče sa rebrima obično se zasniva na zbiru vrednosti koje pripadaju individualnim, odvojenim delovima (prema Timoshenku i Goodieru [116]. Rowe [117] nije uopšte uzeo u obzir efekt Poissonovog broja a Timoshenko i Woinowski — Krieger [61] su uveli Poissonov broj samo u deo poprečnog preseka obrazovanog pločom a ne i u sistem rebara. U većini slučajeva njihova metoda daje vrlo netačne (obično veće) vrednosti. U svojoj teoriji, zasnovanoj na Pflügerovom radu [107] Gienke [108] je uveo Poissonov broj u ceo sistem, ali su njegove naponske jednačine tako formulisane kao da su poprečna rebra u jednakom kontaktu celom dužinom podužnih rebara; uticaj Poissonovog broja bio bi ovako značajno uveličan.

Jackson [118] je predložio izračunavanje torzione krutosti celog poprečnog preseka membranskom analogijom; ploča nije razmatrana kao kontinualni elemenat koji prenosi smičući tok i vrednosti torzione krutosti su tada suviše visoke. Bareš i Machan [111] su uveli torzionu krutost baziranu na interakciji torzione krutosti izotropne ploče i zatvorenog šupljeg profila ukrućenja. Cusens, Zeiden i Pama [119] su modificirali Gienckeov pristup da bi ga prilagodili jednom određenom aranžmanu, ali pri izvođenju naponskih jednačina oni nisu mogli da izbegnu određene netačnosti. Cornelius [100] uvodi tzv. efektivnu torzionu krutost; Hoppmann [120] je procenio krutost na osnovu eksperimenata; Trenks [110] je propustio da radi sa torzionom krutošću greda i ekscentricitetom u poprečnom pravcu itd. Pošto ne egzistira bilo kakva ortotropna ploča (u Huberovom smislu) perfektno ekvivalentna ploča sa asimetričnim ukrućenjima nema opšteg obrasca za modeliranje torzione krutosti Huberove ploče ekvivalentne realnoj rebrastoj ploči; najviše što se može je izvođenje formula ograničenih samo na jedan specijalan problem, i saglasnost može biti postignuta samo kod ugiba a ne i za naponsko stanje.

Uprkos činjenici da je predmet široko i isorpno studiran, tako da bi izgledalo da mu je poklonjena odgovarajuća pažnja, autor ovoga članka oseća da to nije tako. Metode koje koriste računare mogu dati samo rezultate koji su potpuno zavisni od pretpostavki i ulaznih podataka. One u isto vreme odvlače delo, konstrukciju, od neposrednog posmatranja statičara: on teško može da proverava kako se utiče na različite statičke veličine, što u stvari rezultira neekonomično projektovanom konstrukcijom, mada proračun može biti vrlo tačan. Kompjuterska optimizacija je prilično skupa i, šta više, još postoji opasnost lakog previda grešaka u ulaznim i izlaznim podacima. Dokazano je da su neke ulazne vrednosti i pretpostavke prilično neodgovarajuće. S druge strane, aproksimativne metode daju rezultate direktno, i to čine dovoljno brzo, ali pošto pretpostavke koje ih pojednostavljuju nisu potvrđene, one nisu u položaju da daju istinitu belešku o stanju napona u konstrukciji.

Tako smo mi još uvek suočeni sa problemom kako se koncentrisano opterećenje, ili opterećenje, u stvarnosti raspodeljuje među različite članove nekog mostovskog sistema ako se variraju vrednosti poprečne kontrakcije i krutosti na savijanje i torziju. Za projektanta nekog mosta od velikog su značaja sledeće stvari: određivanje optimalne vrednosti poprečne krutosti i, kod prethodno napregnutih konstrukcija, određivanje kritične vrednosti poprečnog prednaprezanja; mogućnost određivanja efekta nekog abnormalnog tereta za koji most nije dimenzioniran i određivanje optimalnog položaja takvog opterećenja na mostu, ili mogućnost selekcije iz izvesnog broja mostova onoga koga bi, sa ekonomske tačke gledišta, bilo najbolje rekonstruisati da bi mogao preneti takvo abnormalno opterećenje. Važno je da projektant bude u mogućnosti da, pod istim početnim pretpostavkama, izvrši proračunavanje različitih stepena tačnosti i lako proveri izlazne podatke iz računara kako bi se izbegle velike greške. Bilo bi neophodno da osnovni proračunski postupak uvodi minimum netačnosti i da svi odlučujući uticaji budu uzeti u račun.

Uzimajući u obzir ove zahteve autor je predložio jednu kompleksnu zatvorenu teoriju za analizu ravnih ortotropnih konstrukcija mostovskog tipa, uključujući dotle zanemarivan efekt poprečne kontrakcije, u obliku koji bi omogućavao i precizno rešenje uz pomoć računara i dovoljno brzo, prosto i lako pregledno približno rešenje (sa slobodnim izborom stepena tačnosti) [121, 122, 123]. Ovaj dvostruki karakter metode je neophodan uslov za uklanjanje svih mana prethodnih rešenja; ona omogućava proveru rezultata izašlih iz računara a isto tako i međurezultata u bilo kojoj fazi proračuna.

Rešenje je zasnovano na autorovim ranijim radovima i koristi analizu naponskog stanja materijalno ortotropnih ploča. Dat je oblik koji omogućava da većina numeričkih proračuna bude izvršena unapred tako da olakšava tabulaciju parcijalnih rezultata (bezdimenzionalnih koeficijenata) u korelaciji sa odlučujućim karakteristikama istraženih konstrukcija (bezdimenzionalni parametri). Tabulisanje bezdimenzionalnih koeficijenata je izvršeno za glavne vrednosti ovih karakteristika [124]; za druge vrednosti karakteristika izvedeni su interpolacioni obrasci na osnovu precizno izračunatih numeričkih vrednosti koeficijenata tako da greška usled interpolacije ne bi prelazila tri procenta u bilo kojem odlučujućem slučaju [125, 126].

Nađen je prelaz sa realne konstrukcije na ekvivalentnu ortotropnu ploču [127], definisana je analogija između konstrukcione i materijalne anizotropije [128] i izabran je podesan postupak za iznalaženje potpuno odgovarajućih vrednosti krutosti na savijanje i torziju sa prikladnim uzimanjem u obzir uticaja poprečne kontrakcije [129].

Ustanovljen je stepen tačnosti rezultujućih veličina, koje su od odlučujuće važnosti za projektovanje konstrukcija. Objašnjen je uticaj različitih pretpostavki na veličinu greške a predložena je i definicija primenljivosti metode [128].

Rezultati metode bi se mogli koristiti u proceni i detaljnoj analizi uticaja dosledno uvedene poprečne kontrakcije konstrukcije i uticaja poprečne kontrakcije uvedene samo u jednom delu (fazi) analize; rezultati analize se takođe mogu koristiti za određivanje veličine greške nastale usled činjenice da je taj uticaj zanemaren.

Put ka daljem uprošćavanju i tačnosti rešavanja ovoga vrlo teškog problema mehanike konstrukcija će se nesumnjivo nastaviti u prkos činjenici da je u naše vreme teško zamisliti bilo kakvo revolucionarno otkriće ili promenu u algoritmu analize. Analiza elasto-plastičnog ili plastičnog stanja ovih konstrukcija mogla bi se smatrati budućim putem punim nade.

(Preveo sa engleskog: dr *Boško Petrović*)

LITERATURA:

[1] Jansonnius G. F., Nieuwe Vereffeningsmethoden voor het berekon van Balkooters, Waltmen, Delft, 1948.
[2] Beer H., Ein baustatischas Vorfahren zur Rechung orthotroper Platten und Plattenroste, Österr. Ing. Arch., 9, 78, 1955.
[3] Jäger K., Österr. Bauzeitschrift, 3, 1953, str. 58.
[4] Handbuch für Eisenbetonbau, Berlin, roč. 6, 1931.
[5] Beer H., Reisinger F., Nachr. der Österr. Betonvereins, 14, 1954, str. 13, Příloha k Österr. Bauzeitschrift č. 3, 9, 1954.
[6] Bleich F., Melan E., Die Differenzengleichungen der Bau-Statik, Perlín, 1924.
Bleich F., Melan E., Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik, Berlín, 1927.

[7] Koch J., Enige toepassingen van de leer der eigenfuncties op vraagstukken uit de toegepaste mechanika, Dalft, 1929.
[8] Thoms A., Die Berechung mehrafach symmetrischer Trägerrosto mit Hilfe von Sinus-Gewichten, Stahlbau, 1936, str. 138.
[9] Melan A., Schindler R., Genaue Berachnung von Trägerrosten, Videň, 1942.
[10] Homber H., Weinmeister J., Einflussflächen für Kreuzwerke, Springer 2. vyd., Berlin, 1956.
Homberg H., Kreuzwerke (Forschungschete aus dem Gebiete des Stahlbaues, č. 3), Berlín, 1951.
Homberg H., Trenks K., Drehsteife Krezwerke, Berlín, 1962.
[11] Homberg H., Weinmeister J., Einflussflächen für Kreuzwerke, Springer Verlag, Berlín, 1956.
[12] Clarkson J., The elastic Analysis of Flat Grillages, Cambridge Eng. S., Gen. Ed. J. Barker, F. R. S. 1965.
[13] Biermann W., Trägerrost Tabellen, VEB Verlag Technik, Berlín, 1969.
[14] Timoshenko S., Über die Biegung von Trägerrosten, Z. f. angew. Mathem. u. Mech. č. 1933.
[15] Hetenyi M., Beams on elastic foundation, Univ. of Michigan Press, 1946.
Hetenyi M., A method of calculating grillage beams, S. Timoshenko 60th Anniversary Volume. New York, 1938, str. 60—72.
[16] Weber E., Berechung zweisaitig gelagertos Trägerroste mit Hilfa von Sinusreihen, Bauingenieur, str. 227—230, 1940.
[17] Hondermarcq H., Etude des tabliers de ponts à dalle nervurée ou à dalle continue reposant sur longerons, Bull, Cours Constr. Génie Civil, č. 4, Liège, 1941.
[18] Ostenfeld C., Lastverteilende Querverbindungen, Jul. Gjellerup, Kodaň, 1930.
[19] Genntner R., Der Eisonbetonträgerrost, Beton u. Eisen, č. 22, 23, 1928.
[20] Palotás L., Iléssy J., Kollár L., Tartórácsok számitása, Budapest, 1953.
[21] Hrennikoff A., Solution of Problems of Elasticity by the Framework Method, J. Appl. Mech. 63, New York, prosinec, 1941.
[22] Newmark N. M., Numerical Methods of Analysis of Bars, Plates and Elastic Bodies, Numerical Method of Analysis in Engineering, The Mac Millan Co., New York, 1949.
[23] Ang. A. H. S., Newmark N. M., A numerical procedure for the analysis of continuous plates. Proc. 2nd ASCE Conference on Electronic Computation, Pittsburgh, 1960.
[24] Yettram S. L., Husain H. M., Grid — Framework Method for Plates in Flexure, J. Eng. Mechn. Div. ASCE, roč. 91, New York, červen 1965.
[25] Christenson R. M., Vibration of a right trianlugar plate by a gridwork method J. Amer. Inst. of Aeronautics and Astronautics, roč. 1, 1964, str. 1790—1795.
[26] Lightfoot E., A grid framework analogy for laterally loaded plates, Int. J. Mech. Sc., roč. 6. Oxford, červen 1964.
[27] Hudson W. R., Matloch H., Cracked Pavement Slabs with Nonuniform Support, J. Highway Div., ASCE, roč. 93, č. HW 1, duben 1967.
[28] Willems N., Lucas W. M., Matrix Analysis for Structural Engineers, Prentice Hall, Englewood Cliffs, New Jersey, 1968.
[29] Gere J. M., Weaver W., Analysis of framed structures, D. van Nostrand Co.
[30] Laursen H. I., Matrix Analysis of Structures, Mc Graw-Hill, New York, 1966.
[31] Rubenstein M. F., Matrix computer analysis of structures, Prentice Hall, Englewood Cliffs, New Jersoy, 1966.
[32] Eisenmann K., Woo L., Namyat S., Space frame analysis by matrices and computer, J. Str. Div., Prec. Paper, roč. 83 ASCE, č. 3365, New York, prosinec, 1962.
[33] Pippard A. J., Waele J. P. A., The loading interconnected bridge girders, Journal ICE 10, č. 1, 1938, str. 97—114.
Pippard A. J., Studies in Elastic Structures, Arnold, Londýn, 1952.
[34] Southwell R. V., Relaxation methods in engineering, University Press, Oxford, 1940.
[35] Gibšman E. E., Gibšman M. E., Teorija i raščot predvaritelno naprjaže nych železobetonnych mostov, Avtotransizdat, Moskva, 1963.
[36] Russelt H. van, Constructions continues à barres droites, Rev. Univ. des Mines, č. 10, str. 461—526, 1947.
[37] Lazarides T. O., The design and analysis of openwork prestressed concrete beam grillaged, Civil Engineering 47, č. 552, str. 471—473, 1952.
[38] Bechyně S., Batonové mosty trámové a rámové, Techn. průvodce 11, Praha, 1954.
[39] Leonhardt F., Die vereinfachte Berechung zweiseitig gelagerter Trägerroste, Bautechnik, str. 535 1938.
Leonhardt F., Ableitung für die verainfachte Trägerrostberechung, Berlín, 1940.
Leonhardt F., Vereinfachte Trägerrostberechung. Nachweis des Genauigkeitsgrades und Erweiterungen, Bautechnik, 1940, str. 86—91, 104—110.
Leonhardt F., Andrä W., Die vereifachte Trägerrostberechung, Stuttgart, 1950.
[40] Pacholík L., Zjednodušen řešení roštových soustav, Praha, 1947.
[41] Brebera V., Výpočet nosínků vroštu, Praha, 1946.
[42] Faltus F., Theorie lastverteilender Querverbindungen, Bauingenieur, č. 46, Teorie příných wýzth. Technický obzor č. 1, 2, 3, 4, 1927.
Faltus F., Lastverteilende Querverbindungen, Zeitschrift des Österr. Ing. u. Arch., č. 43/44, 45/46, 1928.
Faltus F., Zuschriften (à/89/Leonhardt — Andrä), Bautechnik, č. 17 a 28, 1939.
Faltus F., Přibližné řešení nosníkových roštů, Sborník ČVUT, č. 8, Praha, 1949,
[43] Dašek V., Řešení trámových roštů metodou harmonického zatížení, Praha, 1953.
[44] Chwalla E., Über die Grundgleichungen der allgomeinen Orthotropen Scheiben und Platten, Rendiconti e Publicazioni del Corso di Perfezionamente del Politechnico di Milano, č. 7, 1957.
[45] Přesná citace není známa, ale tato metoda je přisuzována Engesserove všeobecně (viz /4/) nebo
Bayer K., Die Statik im Eisenbetonbau, 2. vyd. str. 673, Berlín, 1934.
[46] Courbon M., Calcul des ponts à poutres multiples solidarisées par des entretoises, Ann. des Ponts et Chaussées de France, listopad-prosinec 1940, str. 293—322.
[47] Mallet Ch., Calcul des ponts à poutres multiples, Travaux, listopad 1946, str. 413—422.
[48] Gibšman E. E., Ústaní sdělení, Moskva, 1966.
[49] Proskurjakov L. D., viz Ulickij B. E: Prostransrvonnyje raščot mostov, Transport, Moskva, 1967.
[50] Grigorjev V. V., viz Ulickij B. E.: Prostranstvennyje raščoty mostov, Transport, Moskva, 1967.
[51] Ilijasevič, viz Ulickij R. E.: Prostranstvennyje raščoty mostov, Transport, Moskva 1967.
[52] Gaussens J. E., Étude des ponts à quatre poutres, Travaux, 1948, str. 203.
[53] Trost H., Lastverteilung bei Plattenbalkenbrücken, Düsseldorf, 1961.
[54] Ulickij B. E., Prostranstvennyje raščoty baločnych mostov, Avtotransizdat, Moskva, 1962.
[55] Todhunter J., Pearson K., History of the theory of Elasticity, Dover Publication, Vol. 1, New York.
[56] Kirchhoff G., Vorlesung über mathematische Physik, Mechanic, roč. 1, 1876.
[57] Winkler E., Die Lehre von der Elastizität und Fostigkeit, Praha, 1867.
[58] Zimmermann H., Die Berechnung des Eisenbahnobersbaues, Berlín, 1888, 2. vyd. Berlín, 1930.
[59] Hertz A., Gesammelto Werke, 1895.
[60] Westergaard H., Ingenioren, vol. 32, 1923.
[61] Timoshenko S., Woinowsky-Krieger S., Theory of Plates and Shells, Mc Graw Hill, New York, 1959.
[62] Livesley R. K., Some Notes en the Mathematical Theory of a loaded Elastic Plate Resting on an Elastic Foundation, Quarterly J. of Mathematic, Oxford, březen 1953.
[63] Keer A. D., Elastic Plates on a Liquid Foundation, J. Eng. Mech. Div. ASCE, červen 1963.
[64] Gorbunov-Posadov M. I., Raščot konstrukcij na uprugom osnovanii, GIL po stroj. i arch., Moskva, 1953.
[65] Klepnikov S. N., Raščot konstrukcij na uprugom osnovanii, Izd. Budibelnik, Kijev, 1967.
[66] Korenev B. G., Černigovskaja E. I., Raščot plit na uprugom osnovanii, Gosstrojizdat, Moskva, 1962.
[67] Gorloy A. M., Serebrjanyj R. V., Avtomatisovanyj raščot prjamougolnych plit na uprugom osnovanii, Izd. lit. po stroitelstvu, Moskva, 1968.
[68] Žemočkin B. N., Sinicyn A. P., Praktičeskie metody raščota fundamentnych balok i plit na uprugom osnovanii, Gosstrojizdat, Moskva, 1962.
[69] Hearmon R. F. S., An introduction to applied anisotropic elsaticity, Oxford, 1961.
[70] Lechnickij, S. G., Teorija uprugosti anisotropnovo těla, Gostechizdat, Moskva, 1950.
[71] Ambarcumjan S. A., Teorija anisotropnych plastin, Izd. Nauka, Moskva, 1967.
[72] Huber M. T., Teorie plyt prostokatnio rožnokierunkowych, Lvov, 1921.
Huber M. T., Über die Biegung einer Rechteckplatte von ungleicher Biegungsfestigkeit in der Ländsund Querrichtung, Bauingenieur 6, 1924, str. 259 a 305.
Huber M. T., Über die Biegung oiner sehr langen Eisenbetonplatte, Bauingenieur 7, 1925, str. 7.
Huber M. T., Über die genau Berchung einer orthotroppen Platte, Bauingenieur 7, 1925, str. 878.
Huber M. T., Einige Anwendungen der Biegungstheorie orthotropar Platten, Z. angew. Math. Mech., 1926, str. 229.
Huber M. T., Stereomechanika, techniczna, Varšava, 1947.
Huber M. T., Die Theorie der kreuzweise bewehrten Eisenbetonplatte nebst Anwendungen auf mehrere bautechnisch wichtige Aufgaben über rechteckige Platten, Bauingenieur 4, 1923, str. 845, 392.
[73] Argyris J. H., Energy Teorems and Structural Analysis, Butterworths, Londýn, 1960.
Argyris J. H., Continua and Discontinua, Proc. of the First Conf. on Matrix Methods in Structural Machanics, Ohio, 1965.
[74] Clough R. W., Tocher J. L., Finite Elements Stiffness Matrices for Analysis of Plate Bending, Proc. of the First

Conf. on Matrix Methods in Structural Mechanics, Ohio, 1965.
[75] Turner M. J., Clough R. W., Martin H. C., Topp L. Y., Stiffness and Deflaction Analysis of Complex Structures, Aeron. Sciences, 23, 1956, str. 805—823.
[76] Melosh R. J., Basis of Derivation of Matrices for the Direct Stiffness Method, J. AIAA, 1, 1963. str. 1631.
Melosh R. J., A Stiffness Matrix for the Analysis of Thin Plates in Bending, J. Aeron. Sci., 23, 1961, str. 34.
[77] Zienkiewicz O. C., The Finito Element Method, Mc Graw Hill, New York, 1967.
[78] Mehrain M., Finite Element Analysis of Shew Composite Girder Bridges, Berkeley, 1967 (Diss.)
[79] Guyon Y., Calcul des ponts-dallos. Ann des Ponts et Chaussées de France 119, 1949, str. 555—589, 683—713.
Guyon Y., Calcul des ponts larges à poutres multiples solidarisées par des entretoises. Ann. des Ponts at Chaussées de France, č. 9, 10, 1946, str. 553—612.
Guyon Y., Note en réponse au mémoire de V. A., Nikolsky Ponts et Chassées de France, září-říjen 1952. str. 619—629.
[80] Masselin M., Flexion transversale des ponts à poutres multiples. Annales des Ponts et Chaussées, 1939, str. 541.
[81] Massonnet Ch., Méthode de calcul des ponts à poutres multiples tenant compto de leur résistance à le torsion, Mémoires A. I. P. C. 10, 1950. str. 147—182.
Massonnet Ch., Compléments à la méthode de calcul des ponts à poutres multiples, Ann. Trav. Publ. de Belgique, 107, č. 5, 1954, str. 630—748.
[82] Torroja E., Villena, L., Estudio teoretico fotoelastico de emparillados, Madrid, 1945.
[83] Newmark N. M., A distribution procedure for the analysis of slabs continuous over flexible beams, Bull. Univ. of Illinois, E. E. Station, č. 304, 1938.
Newmark N. M., Siess, C. P. Moments in I beam-bridges. Bull. Univ. of Illinois, E. E. Station, č. 336, 1943.
Newmark N. M., Siess C. P., Penman R. H., Studies of highway bridges, Public Roads, č. 1, 2, 3, 1943, str. 157—164.
Newmark N. M., Sioss C. P., Penman R. H., Studies of slabs-beam highway bridges. Bull. Univ. of Illinois, č. 375, 1948.
[84] Richart F. E., Newmark N. M., Siess C. P., Highway bridge floors, Proc. A. S. C. E., červen 1948.
[85] Reynolds G. C., The strength of right prestressed concrete slabs bridges with edge beams, C. C. A. Technical Report TRA/327 prosinec 1956.
[86] Ritchie C. C., Studies on Bridge-Deck Systems (II. Bending Moments in a Plate and Girder System) National Building Studies Research Paper, č. 31, Londýn, 1960.
[87] Thomas F. G., Research on the strenght of bridges, Mémoirs A. I. C. P., 1949, str. 45—61.
[88] West R., Clark L. A., Bibliography of Experimental Work of Slab and Pseudo-slab Bridges, Dept. Note DN/7004, CCA, Londýn, červen 1971.
[89] Bareš R., Massonnet Ch., Analysis of beam-grids and orthotropic plates by the Guyon-Massonnet-Bareš method, Crosby-Lookwood, Londýn 1968, Drunod, Paříž 1966, Kioritsu Shupan Co., Tokio 1969.
[90] Cusens A. R., Pama R. P., Design curves for the approximate determination of bending moments in orthotropic bridge decks, University of Dundes, Civ. Eng. Dept., listopad 1971.
[91] Krug S., Stein P., Einflussfelder orthogonal anistroper Platten — Influence surfaces of orthogonal anistropic platos, Springer, Berlín, 1961.
[92] Pucher A., Einflussfelder elastischer Platten, Springer, 3. vyd., Berlín, 1964.
[93] Bittner E., Momentemetalfen und Einflussflächen für kreuzweis bewehrte Eisenbetonplatten, Springer, Vídeň, 1938.
[94] Oisen H., Reinitshuber F., Die zweiseitig gestützte Platte, Ernst u. S., Berlín, 1959.
[95] Hoeland G., Stützenmomenten-Einflussfelder durchlaufender Platten, Gringer, Berlín, 1957.
[96] Hájek J., Vpyvové plochy ortotropnej dosky, Bratislava, 1954.
[97] Lechnickij S. G., Anisotropnyje plastinki, Moskva, 1947.
[98] Homberg H., Trenks K., Drahsteife Kreuzwerke, Springer, Berlín, 1962.
[99] Kawai T., Thürlimann B., Influence surface for moments in slabs continuous over flexible cross beams, IVBH — Abhandlung, Bd. 17, 1957.
[100] Cornelius W., Die Berechung der ebenen Flächentragwerke mit Hilfe der Theorie der orthogonal-anisotropen Platte, Stahlbau 21, 1952, str. 21—24, 43—43, 60—63.
[101] Dončenko V. G., Prostranstvennyj rasčot baločnych avtodorožnych mostov, Moskva, 1953.
[102] Sítek O., Výpočet ortotropních desek, VIS, Praha, 1958.
[103] Hossain I., Coefficients to analyse recetangular slab bridges, J. Str. Div., ST 1, leden 1975.
[104] Rowe R. E., A load distribution theory for bridge slabs allowing for the effect of Poisson's ratio. Mag. of Concrete Research, 7, č. 20, 1955, str. 69.
[105] Rostovcev G. G., Calculation of a thin plan sheeting supported by ribs, Trudy Loningr. Inst. Inženerov Grazd. vozdošnovo flota, č. 20, 1940.
[106] Homberg H., Über die Lastverteilung durch Schubkräfte. Theorie des Plattonkreuzwerkes, Stahlbau 1952, str. 42—43, 64—67, 77—79.
[107] Pflüger A., Die orthotrope Platte mit Hohlsteifen, Österr. Ing. Arch. 2, 1955, str. 199.
[108] Giencke E., Die Grundgleichung für die orthotrope Platte mit exzentrischen Stoifen, Stahlbau 25, 1955/6, str. 123—129.
[109] Massonnet Ch., Plaques et coques cylindriques orthotropes à nervures dissymétriques, Mémoires A.I.P.C. 19, 1959, str. 201—230.
[110] Trenks K., Beitrag zur Berechnung orthogonal anisotroper Rechtecklatten, Bauingenieur, 29, č. 10, 1954, str. 372—377.
[111] Bareš R., Machan P., Přesný výpočet tvarově ortotropních desek, Rozpravy ČSAV, TV, č. 4, 1962.
[112] Bareš R., Interaction transversale des constructions à poutres solidares d'une dalle, Acta Technica, ČSAV, č. 4, 1959, č. 2, 1960.
[113] Bareš R., Berka L., General Theory of Composite Orthotropic Plates, ÚTAM ČSAV, 1963.
[114] Berka L., Ortotropní desky se stěnovým účinkem. Diss. ÚTAM ČSAV, Praha, 1964.
[115] Kenning G., Zur genauen Berechung konstruktiv ortotroper Platten, Der Stahlbau, š. 3, 1972, str. 78—86.
[116] Timoshenko S., Goodier J. H., Theory of Elasticity, Mc Graw Hill, New York, 2. vyd., 1951.
[117] Rowe R. E., Concrete bridge design, CR Books, 1962.
[118] Jackson N., Torsional rigidity of concrete bridge decks, Concrete 2, str. 468.
[119] Cusens A. R., Zeidan, M. A., Pama R. P., Elastic rigidities of ribbed plates, Build. Sci, roč. 7, Londýn, 1972. str. 23—32.
[120] Hoppmann W. H., Bending of orthogonally stiffoned plates, J. Appl. Mech. Trans. ASME, 22, 1955, str. 267.
Hoppmann W. H., Proc. Soc. Exp. Stress Analysis, roč. 14, č. 1, 1956.
[121] Bareš R., Method of dimensions less coefficients for analysis of structurally orthotropic plane structures, Part 1, Acta Technica ČSAV, roč. 18, 1973, č. 5, str. 462—493.
[122] Bareš R., Method of dimensions less coefficients for analysis of structurally orthotropic plane structures, Part 2, Acta Technica ČSAV, roč. 18, 1973, č. 6, str. 595—621.
[123] Bareš R., Analysis of structurally orthotropic plane systems, Teorijska i primenjena mehanika 1, 1975, str. 73—85.
[124] Bareš R., Méthode des coefficients sans dimensions pour l'analyse des constructions planes présentant l'orthotropie deforme, Ann. des travaux publics de Belgique, no. 6, 1973-74, str. 367—384.
[125] Bareš R., Functions of Interpolation in the Analysis of Structurally Orthotropic Plane Systems by the Method of Dimensionsless Coefficients, Acta Technica ČSAV č. 3, 1975, str. 339—343.
[126] Bareš R., Einfluss der Querkontraktionen bei der Analyse formorthotroper Flächentragwerke, Strasse — Brücke — Tunnel, vol. 46, č. 6, 1974, str. 149—156.
[127] Bareš R., Analysis of structurally orthotropic bridge deck systems with stiffened outer edges, Acta Technica ČSAV, roč. 19, č. 2, 1974, str. 199—215.
[128] Bareš R., Precision de la methode des coeffisients de repartition et grandeur des erreurs dans l'analyse des constructions planes présentant l'orthotropie de forme, Ann. des travaux publics de Belgique, no. 6, 1973-74, str. 385—393.
[129] Bareš R., Destimmung der Biego — Torsionssteifigkeit von Flächentragwerken unter Berücksichtigung der Querkontraktion, VDI-Z, Band 116, č. 10, 1974, str. 834—839.

The State of the Art in the Analysis of the Elastic Behaviour of Plane Beam Systems

Richard A. B A R E Š

Owing to their advantageous static effect, the plane beam systems get wide application in practice, though the actual distribution of stress in these systems, when they are externally loaded, presents great difficulties due to a high degree of static indeterminacy. Only few structures have been paid equal attention as these systems in the history of mechanics. The bibliography pertaining to this subject is extremely rich; tens of hypotheses, procedures, and methods have been worked out in order to clearify the real action of the individual members and to devide a feasible method of analysis. A large number of dissertations, doctoral theses, and works of research faculties and of the staffs of research institutes all over the world have been dealing with the analysis of these systems. However, only some of these works have attained their objectives. In most of the methods the computational share was so large that they could not be made much use of due either to the shortage of time or to the failure to put the mathematical apparatus to its proper use. And those who were more initiated into the problem sometimes contented themselves with only more or less approximate computations. Such was the end of a number of ambitious methods.

Theories covering the elastic behaviour of plane beam systems can generally be divided into three groups:

a/ theories describing the structure in its real form and covering the behaviour of each member in the structure with the help of a system of redundant /linear, simultaneous/ equations; the number of these theories is very high and they are applicable in direct analysis or analysis of relaxation with the help of computers.

b/ theories that substitute a simpler system of equivalent rigidity for the real cross joining of the main beams. The behaviour of the structure is covered by sets of simultaneous differential equations. Only in the simplest cases can computers be dispensed with.

c/ theories that substitute an elastically equivalent system equally distributed in both directions for the real system of individual members. Analysis can make use of a wide range of mathematical repertory employed in the analysis of the continuum, with and without the help of computers.

Theories a/ and b/ , which can be characterized as bar and rod statics, were worked on from the year 1889, when Engesser published his work, until about the year 1945. The renascence of their development began after 1960 when the new computational technology started to be introduced. The different numerical procedures used until 1947 are duly mentioned in Jansennius' comprehensive work /1/, later contributions can be found in Beer's /2/ and Jäger's /3/ works. They testify to repeated attempts at finding the most precise solution of the problem by means of approximative methods.

In the deformation energy method of analysis the unknowns are forces acting between beams and cross beams /4/, in the case of pin joints, or in rigid joints, they are joint forces and moments in two directions /3/,/5/. The number of the unknowns is n , or 3n , if n is the number of joints. Hilal /18/ showed that even in the case of rigid joints the joint forces can be used as the unknown quantities, acting upon both cross and longitudinal beams as two independent systems of continuous, elastically constrained beams. But the solution calls for a preliminary determination of the fixed points of both the systems of beams.

The solution of 3n equations can be avoided if a system of the external, the so-called own loads can be found under which the grid becomes vertically deflected to a simple shape /the deflection lines of all beams or/and cross beams being similar/. These systems of loads must then enter into combinations so as to form the actual state of loading. The method of own loads is a generalization of the Fourier series method, and is in agreement with the general method of the solution based upon the application of orthogonal functions frequently used in mathematical mechanics. This method was first put forth by Bleich and Melan /6/, who made use of the differential analysis; later it was developed in a different way by Koch /7/ and Thoms /8/; the precise solution is also given by Melan and Schindler /9/, Homberg /10/, Trost /53/ and others. The obvious disadvantage consists in the fact that,as a preliminary, it becomes necessary to determine n systems of own loads. A certain simplification is possible if proportionality is kept between the displacements of the joints and the external forces,acting either only upon the longitudinal beams or upon cross beams.

If the number of cross beams is infinite, the own loads are sinusoidal, which enables a passable analysis procedure, as shown by Timoshenko /14/ for cross beams with supported ends, and by Hetenyi /15/ and Weber /16/ for free-end cross beams. Hondermarcq /17/ proceeded along similar lines, substituting an infinite number of cross beams for the continuous slab resting on parallel beams. Pippard's and De Waele's /33/ method makes use of uniform distribution of transverse rigidity; the authors obtained a system of differential equations, the solution of which is relatively too complex, to be applicable in practice. Homberg /11/, Clarkson /12/, Biermann/13/ and others published tables for some types of grillages based on deformation energy method analysis.

In the slope-deflection method the unknown quantities are vertical displacements and angular deformation of joints. This method was first applied to pin joints by Ostenfeld /18/, for rigid joints the solution of this problem was attempted by Genntner, a feasible solution was also put forth by Paletás /20/.

A practical procedure with the help of both the deformation energy and the slope-deflection methods has been made feasible by computerization, especially if the analysis are arranged in matrix form. In this direction, works can be taken advantage of, which model the elastic continuum by grid beam systems. The first monography in this line was published by Hrennikoff /21/ (a square model of grid with constant Poisson's factor 1/3). For more general plane beam models the solution was derived by Newmark /22/, Ang and Newmark /23/ and Yettram and Hussain /24/. Other particular cases were investigated by Christensen /25/, Lightfoot /26/, and Hudson and Matloch /27/. Willems and Lucas /28/ and others /29/,/30/,/31/ published a matrix form of the equations of angular deformations, for a larger number of members the same was done by Eisenmann et al. /32/ in the year 1962. The organization of the computation can thus easily be arranged in a system, so that programming can immediately be started with. The method again consists in letting each member be affected by unit angular deformation in two directions and by unit displacement in the third direction; the forces called forth by unit strains are the so-called rigidity coefficients from which a rigidity matrix for each member is obtained. Rotating these matrices into a unified system of coordinates we get the matrix of the whole system from the condition of compatibility of deformations in joints. The rotated matrices have been prepared beforehand, and for each member only the numerical values of rigidities and dimensions and the angles of the rotation of the matrix are to be substituted. The precise numerical analysis of the grillage by deformation energy method and the arrangement required by electronic computers are also dealt with, in a number of Ulickij's works, e.g., /54/. Despite the fact that the matrix arrangement possesses the above-mentioned advantages and the fact that quick-working computers are readily available, these methods are suited for the analysis of particular cases rather than for the current practice.

In order to avoid a system of a large number of equations various relaxation methods were devised, in which earlier equations, without being written down, are solved by gradual approximation. The grid is the starting point; its joints are gradually loosened (by deflection and rotation in both directions) in a way suggested by Cross for the analysis of frames. For non-rigid joints such method was worked out by Southwell /34/, Jansonnius /1/ and others, for rigid joints by Van Russelt /36/ and Jansonnius /1/. Lazarides /7/ was able to apply this method in investigating pre-stressed beam grids of various types of support condition.

All methods mentioned must, in view of the time-consuming computations, consist in the numerical determination of the influence surfaces; deriving values for internal forces will, however, be rather difficult and inaccurate, which also fully degrades the alleged high degree of precision of the method.

The solution of the problem was attempted by a number of approximative methods investigating only a part of the grid, substituting idealized members for real ones, and introducing a priori simplifications.

Bechyně /38/, e.g., investigated the grid making use of five moment equations; in his case the cross beams were elastic and they rested on elastic supports like continuous beams.

Leonhardt /39/ chooses a simplified cross joining such that all cross beams are concentrated into one central cross beam of equal rigidity, the transfer being based on experimental observation. The torsional effect has not been included. This idea has also been developed by the Czech authors Pacholík /40/, Brebera /41/, Faltus /42/, and Dašek /43/; the latter two solve the problem using the rigidity of cross beams and longitudinal beams in a fixed ratio, or, vice versa, with equal sections of cross beams and/or longitudinal beams for loading graded according to the development of deflections (so-called ideal configuration of loading or harmonic loads); this enables them again to introduce one, the so-called ideal, cross beam. However, the solution is lacking in clarity and is rather difficult to achieve for multiple states of loading and multiple beam systems.

The methods using a single elastic cross beam were perfected by Kollár /20/, who includes torsion into his consideration.

A further simplification is the replacement of elastic cross beams infinitely rigid cross beams, which is the Engesser method /45/ mentioned above. This method was later recommended by Courbon /46/, Mallet /47/, Gaussens /53/, and according to Gibshman's /48/ oral communication similar approximative solutions (of Proskurjakov from 1902 /49/, Grigorjev /50/, and Iljasevich) are still being made use of in the Soviet designing centres.

Theories under c/, in which the real system of individual members is reduced to an elastically equivalent, continuously spread (smeared, dispersed) system, represent a transition to mechanics of the continuum, by an application of the differential equation of an orthogonally anisotropic slab.

The fundamental differential equation of the deflection of thin slabs was derived by Lagrange and Germain in the year 1811 /55/, Poisson /72/ in his chapter on elasticity (1829) introduced for the first time the investigation of elastic slabs using general elasticity equations; some boundary conditions of his formulation allowed the solution of a few simple cases only. In the year 1850 Kirchhoff /53/ using energy principles derived a describing equation and the corresponding boundary conditions, the number of which on the free edges he reduced. The first study of beams resting upon elastic foundation was published in the year 1867 by Winkler /57/. His theory was amplified by Zimmermann /58/ in 1888. In the field of slabs resting upon elastic foundation the first contribution was by Herz /59/

in the year 1881, who investigated the slab under concentrated load. Herz like Winkler made the assumption for the reaction of the foundations to be in proportion to the deflection of the elastic slab. In the year 1923 Westergaard /60/ extended the theory of the infinite, elastically supported slab so as to get a practically feasible method of analysis of foundation slabs; this method is still in use. Timoshenko and Woinowsky-Krieger /61/ published the most recent discussion of the problem; they dealt with a cetrally loaded circular slab. In 1953 Livesley /62/ arrived at a formal solution of problems concerning the semi-infinite slab and the infinite quadrant. There followed further contributions by a large number of authors, e.g., Kerr /63/, Gorbunov-Posadov /64/, Klepnikov /65/, Korenev and Czernigovska /66/, Gorlov and Serebrjanyj /67/, Zhemochkin and Sinicyn /68/ etc.

Fundamental works concerning the theory of anisotropic slabs came from Huber /72/ in 1921. A survey of the development of the equations and various constants of anisotropy is given by Chwalla /44/, Hearmon /69/, Lekhnicky /70/, Ambarcumyan /71/ and others.

Since the precise analysis of the strain and stress of thin slabs is possible only in special cases, use is made, when dealing with general systems of slabs, of various approximative methods of solution. After the full elaboration of the classic solutions of Lagrange's equation (Navier's, Galerkin's, Levy's, finite-differences etc.), the method of finite elements was developed in the late fifties dealing with arbitrary two-dimensional problems, e.g., by Argyris /73/, Clough /74/,/75/, Melosh /76/, and Zienkiewicz /77/. This method divides the continuum into geometrically simple elements stressed by unknown quantities of internal forces, which are determined from the conditions of geometrical compatibility. For ribbed plates this method was elaborated, e.g., by Mehrain /78/. The computation is relatively quick provided it is perfectly organized. The degree of precision depends upon the form and the size of the selected element and upon the degree of the describing polynomials. Once the programme has been tested the computation of one state of loading of a system of, e.g., 1 000 equations takes about half a minute. Instead of finite plane elements use can be made of the method of (finite) bar (linear) elements, as shown by, e.g., Eisenmann, Woo and Namyet /32/. Both the methods yield comparable results. The method of plane element allows deformations in the joints of the substitute system and the mathematical relationships are best defined by functions of strain. Whereas the method of bar-shaped elements makes use of strainable elements with rigid joints thus coming near the more currently used computational techniques (deformation energy methods). The advantage of this methods is that a change in boudary conditions will not affect either the procedure or the describing equations, and various both external and internal irregularities, like holes, changes in rigidity etc., can easily be included.

It is a disadvantage of these methods, like all methods using computers on a wide scale, that the static analyst is rather detached from the real structure, he cannot check up on the result, select the most economical alternative etc. Since the so-called exact computation (i.e., a multi-digit number pertaining to the described system) is of no use owing to the fact that a number of idealizations must necessarily have entered the initial parameters. The situation here is similar as with theories under a/ and b/, that is also in solving the problems of the continuum various approximative methods find wider practical application. Here too, the methods employing distribution coefficients (as the methods of Leonhardt, Faltus, Dašek etc.) proved to be advantageous.

Guyon /79/, drawing on Masselin /80/, derived the method of distribution coefficients for structures with zero torsional rigidity, Massonnet /81/, using Hetenyi's approach /15/, extended this method to cover torsionally rigid structures. The basic assumption of both these methods is that transverse distribution of any load is the same as that of sinusoidal loads, which is what Dašek /43/ derived for beams of constant cross section. Detailed experimental analysis showed quide good agreement with reality, in spite of the fact that in the distribution of internal forces there were some discrepancies especially in structures with slab, or in prestresses structures (e.g., /82/,/83/,/84/,/85/,/86/, /87/. The reason of it, is that Guyon's assumption of the equal distribution of every load has limited validity, if the influence of transverse contraction is disregarded.

Further development of the approach using distribution coefficients gave rise to the G-M-B method (Guyon-Massonnet-Bareš method /89/), which enabled an easy analysis of internal forces under any load.

In the year 1971 Cusens and Pama /90/ set forth a modified

G-M-B theory , by computing new coefficients for the determination of deflection and of longitudinal moments for a single concentrated load, as the sum of the first nine terms of the series by which the load was approximated. In this way they virtually narrowed the scope of the original method. They introduced the influence of Poisson's coefficient only into the expression for bending moments of the structure with slab (maximum) torsional rigidity.

On the basis of Pucher's method of singularities /92/ Krug and Stein analyzed influence surfaces for a number of rigidity cases in 1961. The method provided the possibility to check on, to design only some concrete cases; the preparation of the computation for the electronic computer took, according to Sattler, several years; a single influence surface required over 5 000 instructions. Influence surfaces are also dealt with by Bittner /93/, Olsen-Reinitzhuber /94/, Homberg /10/, Hoeland /95/. Apart from this there exist a number of solutions of Huber's equation /72/ accompanied by tables /96/,/97/,/98/,/99/ or by formulae /100/,/101/, /102/.

Hossain (in 1975) /103/ suggested extending the G-M-B method to slab bridge structures on intermediate supports by superimposing the effects of isolated reactions computed from the compatibility condition of deflections upon the effects of external loading. He introduced a numerically simpler computation of influence lines by adjusting the arrangement of the computation.

Rowe /104/ and Hossain /103/ pointed at the importance of the influence of Poisson's coefficient, although they did not succeed in introducing it with all consistency.

A number of authors have proved that exact solution of structurally anisotropic systems strickly on the basis of Huber's equation is impossible, since the distances of the neutral axis change from place to place according to local rigidities. The influence of the wall effect may considerably influence above all the state of stress of the top fibres of the structure. Some methods take account of all forces active in the orthotropic slab, the solution, however, leads to partial differential equation of the eighth order, or to a system of two partial equations of the fourth order. Due to the complexity of boundary conditions it is practically impossible to reach the solution /105-115/ and neither can the use of computer bring about a change here. Applying these procedures calls for various simplifications of the assumptions, which more than offsets any contribution in the way of the original higher degree of precision.

Apart from looking for a feasible way of solution of Huber's equation a number of authors have made great efforts at determining as closely as possible the input quantities, especially the flexural and torsional rigidities. Particularly in the development of methods based on the use of computers the disproportion was strongly felt between the precise computations and the unsatisfactorily defined input parameters.

The determination of the torsional rigidity of a ribbed slab was usually based on the sum of the values pertaining to the individual, separate parts, according to Timoshenko and Goodier /116/. Rowe /117/ did not take any account of the effect of Poisson's ratio and Timoshenko and Woinowski-Krieger /61/ included Poisson's ration only into a part of the cross section formed by the slab, not into the system of ribs. In most cases their method yields very inaccurate (usually higher) values. In his theory based on Pflüger's work /107/ Gienke /108/ considered Poisson's ratio for the whole of the system, but his equations of stress are formulated as if the lateral ribs were in equal contact for the whole length of the longitudinal ribs; the influence of Poisson's ratio would thus be considerably exaggerated. Jackson /118/ suggested evaluating torsional rigidity of the whole cross section after a membrane analogy; the slab is not considered as a continuous member transferring shear flow, and the values of torsional rigidity are then too high. Bareš and Machan /111/ introduced rigidity in torsion from the interaction of the rigidity in torsion of an isotropic slab and the closed hollow profile of the stiffeners. Cusens, Zeidan and Pama /119/ modified Giencke's approach to suit a certain system of arrangement, but in deriving the equations of stress they could not avoid certain inaccuracies. Cornelius /100/ introduces the so-called effective rigidity in torsion, Hoppmann /120/ made estimates of rigidity on the basis of experiments, Trenks /110/ failed to deal with rigidity in the torsion of beams and with excentricity in transverse direction etc. As there does not exist any orthotropic slab (in Huber's sense) perfectly equivalent to a slab with asymmetric stiffeners, there is no general formula for the seeming torsional rigidity of Huber's slab equivalent to real ribbed slab; at most, only formulae can be derived each limited only to one special problem, and agreement can be obtained for deflections only, not for the states of stress.

Despite the fact that the subject has been widely and thorougly studied, so that it could seem that it has been pair proper attention to, the author of this paper feels that it has not. The methods making use of computers can only yield results that are fully dependent upon the assumptions and the data of the input; at the same time they withdraw the work, the structure, from the static analyst's immediate scope of observation: he can hardly check up on how the different static quantities are influenced, which results in the fact that though the computations may be very accurate, the structure is designed uneconomically. Computational optimalization is rather expensive and, moreover, there is still the danger of errors both in the input and output data being easily overlooked. Some input values and assumptions also proved to be rather inadequate. The approximative methods, on the other hand, yield direct results and they do it rapidly enough; but since their simplifying assumptions are unjustified, they are not in a position to give a true record of the state of stress in the structure.

So we are still facing the problem of how the concentrated load, or loads, are in reality distributed among the different members of the bridge system if the values of flexural and torsional rigidities and the values of transverse contraction vary. For a bridge designer the following items are of great importance: the determination of the optimum value of transverse rigidity and, with prestressed structures, the determination of the critical value of transverse prestressing; the possibility of evaluating the effects of an abnormal load, for which the bridge has not been dimensioned, and of determining the optimum place of that load on the bridge, or the possibility of selecting from a number of bridges the one, which from the point of view of economy it will be best to reconstructe in order that it can carry such abnormal load. It is important that the designer should be able to carry out computations under the same initial assumptions with varying degrees of precision and to check up easily on the output data of the computer so as to avoid gross errors. It is necessary that the basic computational procedure should involve the minimum of inaccuracies and that all decisive influences be taken account of.

Taking into account these requirements author suggested a complex closed theory for the analysis of plane orthotropic structures of the bridge type, including the so-far neglected effect of transverse contraction in a form that would enable both a precise solution with the help of computers and an approximative solution (with the option of the degree of accuracy) rapid enough, simple, and easily surveyed /121,122 , 123/. This double character of the method is the necessary condition for dealing with all disadvantages of the previous solutions; it enables a check of both the result put out by the computer and the intermediate calculations of any phase.

The solution is based on author's earlier works and makes use of the analysis of the state of stress in materially orthotropic slabs. It has been given a form which enables most of the numerical computations to be carried out beforehand so as to facilitate tabulation of partial results (of non-dimensional coefficients) in correlation with the decisive characteristics of the investigated structure (non-dimensional parameters). The tabulation of non-dimensional coefficients has been carried out for the main values of these characteristics /124/; for other values of the characteristics the necessary interpolative formulae have been determined on the basis of precisely computed numerical values of coefficients so that the error due to interpolation should not exceed three per cent in any decisive case /125 , 126/.

Conditions have been found for the transition from the real structure to an equivalent structurally orthotropic slab /127/, the analogy between the structural and material anisotropy has been defined /128/, and a suitable procedure has been selected for finding fully adequate values of flexural and torsional rigidities with due regard to the influence of transverse contraction /129/.

The degree of precision of the resulting quantities, which are of decisive importance for the structural design, has been established, the influence of the different assumption upon the magnitude of errors has been clarified, and a definition of the applucability of the method has been suggested /128/.

The results of the method could also be used in the evaluation and detailed analysis of the influence of the consistently introduced transverse contraction of the structure and of the influence of transverse contraction introduced into one part (phase) of the analysis only (as suggested by some authors before); the results of the methods have also been made use of in determining the magnitude of the errors caused by the fact that this influence has been neglected.

A way to a further precision and simplification of the solution of this very difficult problem of structural mechanics will indubitably continue in spite of the fact that it is for the time being difficult to imagine any revolutional discovery or a change of analysis algorithms . An analysis of this structures in elasto-plastic or in plastic state might be considered as a hopeful future way.

REFERENCES

/1/ Jansonnius,G.F. : Nieuwe Vereffeningsmethoden voor het bereken van Balkoosters,Waltman,Delft,1948

/2/ Beer, H.: Ein baustatisches Verfahren zur Rechnung orthotroper Platten und Plattenroste, Österr. Ing.Arch., 9,78, 1955

/3/ Jäger, K.:Österr. Bauzeitschrift, 8,1953, str. 58

/4/ Handbuch für Eisenbetonbau, Berlin, roč. 6, 1931

/5/ Beer, H., Reisinger,F.: Nachr. der Österr. Betonvereins, 14, 1954, str. 13, Příloha k Österr.Bauzeitschrift č. 3, 9, 1954

/6/ Bleich,F., Melan E.: Die Differenzengleichungen der Bau-Statik, Berlín, 1924

Bleich,F., Melan,E.: Die gewöhnlichen und partiellen Differenzengleichungen der Baustatik, Berlín, 1927

/7/ Koch, J.: Enige toepassingen van de leer der eigenfuncties op vraagstukken uit de toegepaste mechanika, Delft, 1929

/8/ Thoms, A.: Die Berechnung mehrfach symmetrischer Trägerroste mit Hilfe von Sinus-Gewichten, Stahlbau, 1936, str.138

/9/ Melan, A., Schindler, R.: Genaue Berechnung von Trägerrosten, Vídeň, 1942

/10/ Homberg, H., Weinmeister, J.: Einflussflächen für Kreuzwerke, Springer 2.vyd., Berlin 1956

Homberg, H.: Kreuzwerke (Forschungshefte aus dem Gebiete des Stahlbaues, č. 8), Berlín, 1951

Homberg, H., Trenks, K.: Drehsteife Kreuzwerke,Berlín,1962

/11/ Homberg, H., Weinmeister J.: Einflussflächen für Kreuzwerke, Springer Verlag, Berlín, 1956

/12/ Clarkson, J.: The elastic Analysis of Flat Grillages, Cambridge Eng.S., Gen.Ed. J.Barker, F.R.S. 1965

/13/ Biermann, W.: Trägerrost Tabellen, VEB Verlag Technik, Berlín, 1969

/14/ Timoshenko, S.: Über die Biegung von Trägerrosten, Z.f.angew.Mathem.u.Mech. č. 4,1933

/15/ Hetenyi, M.: Beams on elastic foundation, Univ.of Michigan Press, 1946

Hetenyi, M.: A method of calculating grillage beams, S.Timoshenko 60th Anniversary Volume, New York, 1938, str. 60-72

/16/ Weber, E.: Berechnung zweiseitig gelagertes Trägerroste mit Hilfe von Sinusreihen, Bauingenieur, str. 227-230, 1940

/17/ Hondermarcq, H.: Etude des tabliers de ponts à dalle nervurée ou à dalle continue reposant sur longerons, Bull.Cours Constr.Génie Civil, č. 4, Liège, 1941

/18/ Ostenfeld, C.:Lastverteilende Querverbindungen, Jul. Gjellerup, Kodaň, 1930

/19/ Genntner, R.: Der Eisenbetonträgerrost, Beton u.Eisen, č.22,23, 1928

/20/ Palotás, L., Illéssy, J., Kollár, L.: Tartóráesek számitása, Budapest, 1953

/21/ Hrennikoff, A.: Solution of Problems of Elasticity by the Framework Method, J.Appl. Mech. 63, New York, prosinec 1941

/22/ Newmark, N.M.: Numerical Methods of Analysis of Bars, Plates and Elastic Bodies, Numerical Method of Analysis in Engineering, The Mac Millan Co, New York, 1949

/23/ Ang, A.H.S., Newmark, N.M.: A numerical procedure for the analysis of continuous plates. Proc. 2nd ASCE Conference on Electronic Computation, Pittsburgh, 1960

/24/ Yettram, S.L., Husain H.M.: Grid-Framework Method for Plates in Flexure, J.Eng.Mechn.Div. ASCE, roč. 91, New York, červen 1965

/25/ Christensen, R.M.: Vibration of a right triangular plate by a gridwork method, J.Amer. Inst. of Aeronautics and Astronautics, roč. 1, 1964, str. 1790-1795

/26/ Lightfoot, E.: A grid framework analogy for laterally loaded plates, Int. J.Mech.Sc., roč. 6, Oxford, červen 1964

/27/ Hudson W.R., Matloch, H.: Cracked Pavement Slabs with Nonuniform Support, J.Highway Div., ASCE, roč. 93, č. HW 1, duben 1967

/28/ Willems N., Lucas W.M.: Matrix Analysis for Structural Engineers, Prentice Hall, Englewood Cliffs, New Jersey, 1968

/29/ Gere, J.M., Weaver, W.: Analysis of framed structures, D.van Nostrand Co,

/30/ Laursen, H.I.: Matrix Analysis of Structures, Mc Graw-Hill, New York, 1966

/31/ Rubenstein, M.F.: Matrix computer analysis of structures, Prentice Hall, Englewood Cliffs, New Jersey, 1966

/32/ Eisenmann, K., Woo, L., Namyet, S.: Space frame analysis by matrices and computer, J.Str.Div., Proc. Paper, roč. 88 ASCE, č. 3365, New York, prosinec 1962

/33/ Pippard, A.J., De Waele, J.P.A.: The loading interconnected bridge girders, Journal ICE 10, č. 1, 1938, str. 97-114

Pippard, A.J.: Studies in Elastic Structures, Arnold, Londýn, 1952

/34/ Southwell, R.V.: Relaxation methods in engineering, University Press, Oxford, 1940

/35/ Gibšman, E.E., Gibšman, M.E.: Teorija i rasčot predvaritelno naprjažennych železobetonnych mostov, Avtotransizdat, Moskva, 1963

/36/ Russelt, H. van : Constructions continues à barres droites, Rev.Univ.des Mines, č. 10, str. 461-526, 1947

/37/ Lazarides, T.O. : The design and analysis of openwork prestressed concrete beam grillaged, Civil Engineering 47, č. 552, str.471-473, 1952

/38/ Bechyně, S.: Betonové mosty trámové a rámové, Techn. průvodce 11, Praha, 1954

/39/ Leonhardt, F.: Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste, Bautechnik, str. 535 1938

Leonhardt, F.: Ableitung für die vereinfachte Trägerrostberechnung, Berlín, 1940

Leonhardt, F.: Vereinfachte Trägerrostberechnung. Nachweis des Genauigkeitsgrades und Erweiterungen, Bautechnik, 1940, str.86-91, 104-110

Leonhardt, F., Andrä, W.: Die vereifachte Trägerrostberechnung, Stuttgart, 1950

/40/ Pacholík, L.: Zjednodušen řešení roštových soustav, Praha, 1947

/41/ Brebera, V.: Výpočet nosníků roštu, Praha, 1946

/42/ Faltus, F.: Theorie lastverteilender Querverbindungen, Bauingenieur, č. 46,Teorie příčných výztuh. Technický obzor č.1,2,3,4, 1927

Faltus, F.: Lastverteilende Querverbindungen, Zeitschrift des Österr.Ing.u.Arch., č.43/44, 45/46, 1928

Faltus, F.: Zuschriften (à/39/Leonhardt-Andrä), Bau - technik, č. 17 a 28, 1939

Faltus, F.: Přibližné řešení nosníkových roštů, Sborník ČVUT, č. 8, Praha, 1949

/43/ Dašek, V.: Řešení trámových roštů metodou harmonického zatížení, Praha, 1953

/44/ Chwalla, E.: Über die Grundgleichungen der allgemeinen Orthotropen Scheiben und Platten, Rendiconti e Publicazioni del Corso di Perfezionamente del Politecnico di Milano, č. 7, 1957

/45/ Přesná citace není známa, ale tato metoda je přisuzována Engesserovi všeobecně (viz /4/) nebo
Bayer, K.: Die Statik im Eisenbetonbau, 2.vyd. str. 673, Berlín, 1934

/46/ Courbon, M.: Calcul des ponts à poutres multiples solidarisées par des entretoises, Ann.des Ponts et Chaussées de France, listopad-prosinec 1940, str. 293-322

/47/ Mallet, Ch.: Calcul des ponts à poutres multiples, Travaux, listopad 1946, str. 413-422

/48/ Gibšman, E.E.: Ústní sdělení, Moskva, 1966

/49/ Proskurjakov, L.D.: viz Ulickij, B.E : Prostransrvennyje rasčoty mostov, Transport, Moskva, 1967

/50/ Grigorjev, V.V.: viz Ulickij, B.E.: Prostranstvennyje rasčoty mostov, Transport, Moskva, 1967

/51/ Iljasevič : viz Ulickij, B.E.: Prostranstvennyje rasčoty mostov, Transport, Moskva 1967

/52/ Gaussens, J.E.: Étude des ponts à quatre poutres, Travaux, 1948, str. 203

/53/ Trost, H.: Lastverteilung bei Plattenbalkenbrücken, Düsseldorf, 1961

/54/ Ulickij, B.E.: Prostranstvennyje rasčoty baločnych mostov, Avtotransizdat, Moskva, 1962

/55/ Todhunter, J., Pearson, K.; History of the theory of Elasticity, Dover Publication, Vol. 1, New York

/56/ Kirchhoff, G.: Vorlesung über mathematische Physik, Mechanic, roč. I, 1876

/57/ Winkler, E.: Die Lehre von der Elastizität und Festigkeit, Praha, 1867

/58/ Zimmermann, H.: Die Berechnung des Eisenbahnobersbaues, Berlín, 1888, 2.vyd. Berlín, 1930
/59/ Hertz, A.: Gesammelte Werke, 1895
/60/ Westergaard, H. : Ingenioren, vol. 32, 1923
/61/ Timoshenko, S., Woinowsky-Krieger, S.: Theory of Plates and Shells, Mc Graw Hill, New York, 1959
/62/ Livesley, R.K.: Some Notes on the Mathematical Theory of a loaded Elastic Plate Resting on an Elastic Foundation, Quarterly J.of Mathematic, Oxford, březen 1953
/63/ Keer, A.D. : Elastic Plates on a Liquid Foundation, J.Eng.Mech.Div.ASCE, červen 1963
/64/ Gorbunov-Posadov, M.I.: Rasčot konstrukcij na uprugom osnovanii, GIL po stroj.i arch.,Moskva,1953
/65/ Klepnikov, S.N.: Rasčot konstrukcij na uprugom osnovanii, Izd. Budibelnik, Kijev, 1967
/66/ Korenev, B.G., Černigovskaja, E.I. : Rasčot plit na uprugom osnovanii, Gosstrojizdat, Moskva,1962
/67/ Gorlov, A.M., Serebrjanyj, R.V.: Avtomatisovanyj rasčot prjamougolnych plit na uprugom osnovanii, Izd.lit.po stroitelstvu,Moskva, 1968
/68/ Žemočkin, B.N., Sinicyn, A.P.: Praktičeskie metody rasčota fundamentnych balok i plit na uprugom osnovanii, Gosstrojizdat, Moskva,1962
/69/ Hearmon, R.F.S.: An introduction to applied anisotropic elsaticity, Oxford University Press, Oxford, 1961
/70/ Lechnickij, S.G.: Teorija uprugosti anisotropnovo těla, Gostechizdat, Moskva, 1950
/71/ Ambarcumjan, S.A.: Teorija anisotropnych plastin, Izd. Nauka, Moskva, 1967
/72/ Huber, M.T.: Teorie plyt prostokatnie rožnokierunkowych, Lvov, 1921
Huber, M.T.: Über die Biegung einer Rechteckplatte von ungleicher Biegungsfestigkeit in der Länds- und Querrichtung, Bauingenieur 6, 1924, str. 259 a 305

Huber, M.T.: Über die Biegung einer sehr langen Eisenbetonplatte, Bauingenieur 7,1925, atr.7

Huber, M.T.: Über die genau Berechnung einer orthotroppen Platte, Bauingenieur 7, 1925, str. 878

Huber, M.T.: Einige Anwendungen der Biegungstheorie orthotroper Platten, Z.angew.Math.Mech., 1926, str. 229

Huber, M.T.: Stereomechanika techniczna, Varšava, 1947

Huber, M.T.: Die Theorie der kreuzweise bewehrten Eisenbetonplatte nebst Anwendungen auf mehrere bautechnisch wichtige Aufgaben über rechteckige Platten,Bauingenieur 4,1923,str.354,392

/73/ Argyris, J.H.: Energy Teorems and Structural Analysis, Butterworths, Londýn, 1960

Argyris, J.H.: Continua and Discontinua, Proc. of the First Conf. on Matrix Methods in Structural Mechanics, Ohio, 1965

/74/ Clough, R.W., Tocher, J.L.: Finite Elements Stiffness Matrices for Analysis of Plate Bending, Proc.of the First Conf. on Matrix Methods in Structural Mechanics, Ohio, 1965

/75/ Turner, M.J., Clough, R.W., Martin, H.C., Topp, L.Y.: Stiffness and Deflection Analysis of Complex Structures, Aeron.Sciences, 23, 1956, str. 805-823

/76/ Melosh, R.J.: Basis of Derivation of Matrices for the Direct Stiffness Method, J.AIAA, 1, 1963, str. 1631

Melosh, R.J.: A Stiffness Matrix for the Analysis of Thin Plates in Bending, J.Aeron.Sci., 28, 1961, str. 34

/77/ Ziehkiewicz, O.C.: The Finite Element Method, Mc Graw Hill, New York, 1967

/78/ Mehrain, M.: Finite Element Analysis of Skew Composite Girder Bridges, Berkeley, 1967 (Diss.)

/79/ Guyon, Y.: Calcul des ponts-dalles. Ann des Ponts et Chaussées de France 119, 1949, str.555-589, 683-713

Guyon, Y.: Calcul des ponts larges à poutres multiples solidarisées par des entretoises. Ann. des Ponts at Chaussées de France, č. 9,10, 1946, str. 553-612

Guyon, Y.: Note en réponse au mémoire de V.A.Nikolský, Ponts et Chassées de France, září-říjen 1952, str. 619-629

/80/ Masselin, M.: Flexion transversale des ponts à poutres multiples, Annales des Ponst et Chaussées, 1939, str. 541

/81/ Massonnet, Ch.: Méthode de calcul des ponts à poutres multiples tenant compte de leur résistance à le torsion, Mémoires A.I.P.C. 10, 1950, str. 147-182

Massonnet, Ch.: Compléments à la méthode de calcul des ponts à poutres multiples, Ann.Trav.Publ. de Belgique, 107, č. 5, 1954, str. 680-748

/82/ Torroja, E., Villena, L.: Estudio teoretico fotoelastico de emparillados, Madrid, 1945

/83/ Newmark, N.M.: A distribution procedure for the analysis of slabs continuous over flexible beams, Bull. Univ. of Illinois, E.E.Station, č.304, 1938

Newmark, N.M., Siess, C.P.:Moments in I beam-bridges.Bull. Univ. of Illinois,E.E.Station, č.336, 1943

Newmark, N.M., Siess, C.P.: Design of slab and stringer highway bridges, Public Roads, č.1,2,3, 1943, str. 157-164

Newmark, N.M., Siess, C.P., Penman, R.H.: Studies of slabs-beam highway bridges. Bull.Univ. of Illinois, č. 375, 1948

/84/ Richart, F.E., Newmark, N.M., Siess, C.P.: Highway bridge floors, Proc.A.S.C.E. červen 1948

/85/ Reynolds, G.C.: The strength of right prestressed concrete slabs bridges with edge beams, C.C.A. Technical Report TRA/327 prosinec 1956

/86/ Ritchie, C.C.: Studies on Bridge-Deck Systems (II. Bending Moments in a Plate and Girder System) National Building Studies Research Paper, č. 31, Londýn, 1960

/87/ Thomas, F.G.: Research on the strenght of bridges, Mémoirs A.I.C.P., 1949, str. 45-61

/88/ West, R., Clark, L.A.: Bibliography of Experimental Work of Slab and Pseudo-slab Bridges, Dept.Note DN/7004, CCA, Londýn, červen 1971

/89/ Bareš, R., Massonnet, Ch.: Analysis of beam-grids and orthotropic plates by the Guyon-Massonnet-Bareš method, Crosby-Lookwood, Londýn 1968, Drunod, Paříž 1966, Kioritsu Shupan Co., Tokio 1969

/90/ Cusens, A.R., Pama, R.P.: Design curves for the approximate determination of bending moments in orthotropic bridge decks, University of Dundee, Civ.Eng.Dept., listopad 1971

/91/ Krug, S., Stein, P.: Einflussfelder orthogonal anisotroper Platten - Influence surfaces of orthogonal anisotropic plates, Springer, Berlín, 1961

/92/ Pucher, A.: Einflussfelder elastischer Platten, Springer, 3. vyd., Berlín, 1964

/93/ Bittner, E.: Momententafeln und Einflussflächen für kreuzweis bewehrte Eisenbetonplatten, Springer, Vídeň, 1938

/94/ Olsen, H., Reinitshuber, F.: Die zweiseitig gestützte Platte, Ernst u.S., Berlín, 1959

/95/ Hoeland, G.: Stützenmomenten-Einflussfelder durchlaufender Platten, Gringer, Berlín, 1957

/96/ Hájek, J.: Vplyvové plochy ortotropnej dosky, Bratislava, 1954

/97/ Lechnickij, S.G.: Anisotropnyje plastinki, Moskva,1947

/98/ Homberg, H., Trenks, K.: Drehsteife Kreuzwerke, Springer, Berlín, 1962

/99/ Kawai, T., Thürlimann, B.: Influence surface for moments in slabs continuous over flexible cross beams, IVBH-Abhandlung , Bd.17, 1957

/100/ Cornelius, W.: Die Berechnung der ebenen Flächentragwerke mit Hilfe der Theorie der orthogonal-anisotropen Platte, Stahlbau 21, 1952, str.21-24,43-48,60-63

/101/ Dončenko, V.G.: Prostranstvennyj rasčet baločnych avtodorožnych mostov, Moskva, 1953

/102/ Sítek, O.: Výpočet ortotropních desek, VTS, Praha, 1958

/103/ Hossain, I.: Coefficients to analyse rectangular slab bridges, J.Str.Div., ST 1, leden 1975

/104/ Rowe, R.E.: A load distribution theory for bridge slabs allowing for the effect of Poisson's ratio. Mag. of Concrete Research, 7,č.20,1955, str.69

/105/ Rostovcev, G.G.: Calculation of a thin plan sheeting supported by ribs, Trudy Leningr.Inst.Inženerov Grazd.vozdušnovo flota,č. 20, 1940

/106/ Homberg, H.: Über die Lastverteilung durch Schubkräfte. Theorie des Plattenkreuzwerkes. Stahlbau 1952, str. 42-43, 64-67, 77-79

/107/ Pflüger, A.: Die orthotrope Platte mit Hohlsteifen, Österr.Ing.Arch. 2, 1955, str. 199

/108/ Giencke, E.: Die Grundgleichung für die orthotrope Platte mit exzentrischen Steifen, Stahlbau 25, 1955/6, str. 123-129

/109/ Massonnet, Ch.: Plaques et coques cylindriques orthotropes à nervures dissymétriques, Mémoires A.I.P.C. 19, 1959, str. 201-230

/110/ Trenks, K.: Beitrag zur Berechnung orthogonal anisotroper Rechteckplatten, Bauingenieur, 29, č. 10, 1954, str. 372-377

/111/ Bareš, R.; Machan, P.: Přesný výpočet tvarově ortotropních desek, Rozpravy ČSAV, TV, č. 4, 1962

/112/ Bareš, R.: Interaction transversale des constructions à poutres solidares d'une dalle, Acta Technica, ČSAV, č. 4, 1959, č. 2, 1960

/113/ Bareš, R., Berka, L.: General Theory of Composite Orthotropic Plates, ÚTAM ČSAV, 1963

/114/ Berka, L.: Ortotropní desky se stěnovým účinkem. Diss. ÚTAM ČSAV, Praha, 1964

/115/ Henning, G.: Zur genauen Berechnung konstruktiv ortotroper Platten, Der Stahlbau, š. 3, 1972, str. 78-86

/116/ Timoshenko, S., Goodier, J.H.: Theory of Elasticity, Mc Graw Hill, New York, 2.vyd., 1951

/117/ Rowe, R.E.: Concrete bridge design, CR Books, 1962

/118/ Jackson, N.: Torsional rigidity of concrete bridge decks, Concrete 2, 1968, str. 468

/119/ Cusens, A.R., Zeidan, M.A., Pama, R.P.: Elastic rigidities of ribbed plates, Build.Sci, roč.7, Londýn, 1972, str. 23-32

/120/ Hoppmann, W.H.: Bending of orthogonally stiffened plates, J.Appl.Mech.Trans. ASME, 22, 1955, str. 267

Hoppmann, W.H.: Proc.Soc.Exp.Stress Analysis, roč.14, č. 1, 1956

/121/ Bareš, R.: Method of dimensions less coefficients for analysis of structurally orthotropic plane structures, Part 1, Acta Technica ČSAV, roč. 18, 1973, č. 5, str. 462-493

/122/ Bareš, R.: Method of dimensions less coefficients for analysis of structurally orthotropic plane structures, Part 2, Acta Technica ČSAV, roč. 18, 1973, č. 6, str. 595-621

/123/ Bareš, R.: Analysis of structurally orthotropic plane systems, Teorijska i primenjena mehanika 1, 1975, str. 73-85

/124/ Bareš R.: Méthode des coefficients sans dimensions pour l´analyse des constructions planes présentant l´orthotropie deforme, Ann.des travaux publics de Belgique, no.6, 1973-74, str. 367-384

/125/ Bareš, R.: Functions of Interpolation in the Analysis of Structurally Orthotropic Plane Systems by the Method of Dimensionsless Coefficients, Acta Technica ČSAV č. 3, 1975, str. 339-343

/126/ Bareš, R.: Einfluss der Querkontraktionen bei der Analyse formorthotroper Flächentragwerke, Strasse-Brücke-Tunnel, vol. 46, č. 6, 1974, str. 149-156

/127/ Barše, R.: Analysis of structurally orthotropic bridge deck systems with stiffened outer edges, Acta Technica ČSAV, roč. 19, č. 2,1974, str. 199-215

/128/ Bareš, R.: Precision de la methode des coeffisients de repartition et grandeur des erreurs dans l´analyse des constructions planes présentant l´orthotropie de forme, Ann. des travaux publics de Belgique, no.6, 1973-74, str.385-393

/129/ Bareš, R.: Bestimmung der Biege- Torsionssteifigkeit von Flächentragwerken unter Berücksichtigung der Querkontraktion, VDI-Z, Band 116, č. 10, 1974, str. 834-839